Analysis of Combined Free and Forced Convection Film Boiling Part II: Combined Free and Forced Convection Region

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INTRODUCTION

In Part I (Nakayama and Koyama) the solutions for the pure forced convection region and the pure free convection region were obtained as the asymptotes to the proposed integral formulation. Excellent agreement with the exact solutions (Cess and Sparrow, 1961; Koh, 1962) has been confirmed under both asymptotic conditions.

It would be natural to expect the present integral formulation to give correct results also for the situation where both the buoyancy force and the free convection play equally important roles in driving the fluid. Solutions for such a mixed convection region have been obtained, and have been compared with the asymptotes derived in Part I.

ITERATIVE SOLUTION PROCEDURE

Using the same notation as in Part I, the final set of the expressions derived in the preceding discussion runs as follows:

$$Nux/(Rex\ Pr\ R/4\ H)^{1/2}(\mu_f/\mu) = \left[u_i^*\left(1+\frac{\Lambda}{3}\right)\Big/ I\right]^{1/2} \tag{1a}$$

and

$$Nux/(Grx\ Pr/16\ H)^{1/4} = \left[2\left(1+\frac{\Lambda}{3}\right)\Big/ \Lambda\, I\right]^{1/4} \tag{1b}$$

The characteristic equations for the shape factors u_i^* and Λ are

$$u_i^{*2} = \frac{2\,x^*}{\Lambda\left(1+\dfrac{\Lambda}{3}\right)} I \tag{2a}$$

and

$$\frac{H}{Pr\,R} = \frac{15}{4}\,\frac{u_i^*\,(1-\Lambda)^2\,I_f\,x^*}{(1-u_i^*)^2\,(1+\frac{3}{2}u_i^*)\,\Lambda} \tag{2b}$$

where

$$I = \frac{\int_0^{x^*}(3+\Lambda)\,u_i^*\,dx^*}{(3+\Lambda)\,u_i^*\,x^*} \tag{3a}$$

and

$$I_f = \frac{\int_0^{x^*}(1-u_i^*)^2\left(1+\dfrac{3}{2}u_i^*\right)dx^*}{(1-u_i^*)^2\left(1+\dfrac{3}{2}u_i^*\right)x^*} \tag{3b}$$

The characteristic equation, Eq. 8a in Part I, has been transformed into Eq. 2b here by eliminating the function I in favor of x^*. Thus, the solution of the problem has eventually been reduced to the determination of the two unknown shape factors $u_i^*(x^*)$ and $\Lambda(x^*)$ from the pair of characteristic equations 2a and 2b. Since Eqs. 3a and 3b for the functions I and I_f also involve the unknowns u_i^* and Λ, the characteristic equations 2a and 2b are implicit in both u_i^* and Λ, and the determination of these unknowns requires an iterative procedure at each integration step. These simultaneous equations can be solved following a procedure similar to the one successfully employed for the problem of laminar and turbulent film condensations (Nakayama and Koyama, 1984a,b).

One may fix u_i^* at each end of the integration step, and find Λ there by iterating on Eq. 2a using a standard scheme such as the Newton-Raphson method. The functions I and I_f must be evaluated for each guess, assuming a linear variation between the value guessed at the end of the integration step and the value determined during the preceding integration step, corresponding to the value at the beginning of the current integration step. Then, the resulting pair of values u_i^* and Λ are substituted back into the other characteristic equation, 2b, and the residual is evaluated to check if the estimated set of the values satisfies the equation. The sequence is repeated by successively making a better guess of u_i^*. In this way the local values of the unknowns $u_i^*(x^*)$ and $\Lambda(x^*)$ are determined within a desired accuracy before moving one step further. The boundary values of the unknowns at $x^* = 0$, needed for the initiation of the integration, may readily be provided by Eqs. 11 and 12 of Part I for the pure forced convection results.

DETAIL OF INTEGRATION

Jacobs and Boehm (1970) reported that some difficulties arise as the interfacial velocity approaches the free stream velocity, i.e., $u_i^* \to 1$. They found that the liquid boundary layer thickness becomes negative when the interfacial velocity exceeds the free stream velocity, i.e., $u_i^* > 1$, which is of course physically impossible. Jacobs and Boehm attributed all these difficulties to the neglect of the inertia term in the vapor phase. As already proved in Part I, these unrealistic results for the body force dominating region are not inherent in the neglect of the inertia term. On the contrary, the negligible inertia is indeed a very reasonable assumption when the vapor film is thin (i.e., $H/Pr << 1$), whether the body force is present or not. As pointed out by Fujii et al. (1971), all the difficulties encountered in Jacobs and Boehm's analysis stem from the incorrect boundary condition imposed on the normal velocity component along the liquid boundary layer edge (they set $v = 0$ instead of $\partial u/\partial y = 0$).

It will be shown below that there are no special difficulties in carrying out the integration starting from the pure forced convection region and moving towards the pure free convection region through the mixed convection region where both the body force and forced convection are equally important. When the dimensionless interfacial velocity u_i^* approaches unity in the mixed convection region, both the denominator and numerator of the righthand side of characteristic equation 2b vanish. Hence, the singularity does exist where $u_i^* = 1$, but it can be removed by perturbing Eqs. 2a and 2b around $x^* = x_{tr}^*$ where both u_i^* and Λ simultaneously approach unity. The results from the perturbation are

$$u_i^* = 1 + \frac{1}{2}(x^* - x_{tr}^*) + o((x^* - x_{tr}^*)^2) \qquad (4a)$$

and

$$\Lambda = 1 + o((x^* - x_{tr}^*)^2) \qquad (4b)$$

where x_{tr}^* should be determined from the following implicit equation as a result of Eq. 2a:

$$\int_0^{x_{tr}^*} (3 + \Lambda)\, u_i^*\, dx^* = \frac{8}{3} \qquad (5)$$

Thus, the aforementioned step-wise iterative calculation must be performed by continuously checking Eq. 5, so that the integration around $x^* = x_{tr}^*$ can be carried out analytically using Eq. 4.

A reasonable estimation for x_{tr}^* may be made substituting the asymptotic results for the pure forced convection region (Eq. 11 in Part I) into Eq. 5, which then yields

$$x_{tr}^* \cong \frac{3}{2} u_i^{*2} \left[\left(1 + \frac{32}{27\, u_i^{*3}} \right)^{1/2} - 1 \right] \Bigg|_{x^* = 0} \qquad (6)$$

The above equation indicates $x_{tr}^* \cong 0.72$ when $u_i^* \cong 1$, and naturally the location of the singularity becomes fairly insensitive for large H/PrR. The actual numerical integration reveals $x_{tr}^* = 0.776$ for $H/PrR = 10^2$ [$u_i^*(0) = 0.925$] and $x_{tr}^* = 0.756$ for $H/PrR = 10^4$ [$u_i^*(0) = 0.991$].

CALCULATION RESULTS

Iterative calculations were carried out for $H/PrR = 10^2$ and 10^4 over the range of $0 \leqq x^* \leqq 10^3$; the results are plotted in Figures 1 through 3. Figure 1 clearly shows that the interfacial velocity u_i^* increases downstream as the buoyancy force accelerates the vapor flow. For the forced convection dominating region, the interfacial velocity is less than the free stream velocity, and the vapor retards the liquid flow. Hence, the interfacial velocity is lower for the larger vapor viscosity (i.e., the smaller H/PrR). As the vapor flow enters into the buoyancy dominating region, the interfacial velocity becomes higher than the free stream velocity. Then, the vapor drags the liquid layer,

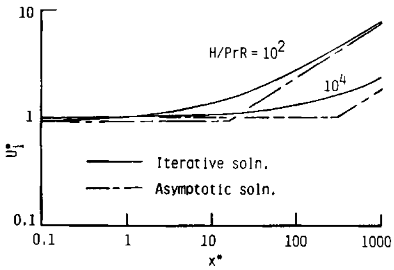

Figure 1. Variation of interfacial velocity.

and naturally, the interfacial velocity is lower for the larger liquid viscosity (i.e., the larger H/PrR).

Equation 14a in Part I indicates that the dimensionless interfacial velocity u_i^* (which is constant for the pure forced convection region) increases in proportion to $x^{*1/2}$ as the buoyancy force becomes predominant. The corresponding development of the velocity profiles is indicated in Figure 2. The profile at $x^* = 0.01$, exhibiting almost linear velocity increase across the vapor layer, represents a typical velocity distribution in the forced convection dominating region. At $x^* = 1$, the vapor velocity maximum can be seen below the liquid-vapor interface ($\eta = 0.986$), and the interfacial velocity slightly exceeds the external free stream velocity ($u_i^* = 1.028$). Farther downstream, the buoyancy force drives the vapor flow, and the strong flow acceleration takes place in the free convection dominating region.

The heat transfer results for $H/PrR = 10^2$ and 10^4 are presented in Figures 3a and 3b, using different heat transfer groupings, one appropriate for the pure forced convection region, the other for the pure free convection region.

The asymptotes for $x^* \ll 1$ and $x^* \gg 1$ can readily be obtained by substituting Eqs. 11 and 14 of Part I into Eq. 1 given here. These can be written as

$$Nux/(RexPrR/4H)^{1/2}(\mu_f/\mu) \cong \begin{cases} u_i^{*1/2} & \text{for } x^* \ll 1 \\ \left(\dfrac{\Lambda + 3}{\Lambda} x^* \right)^{1/4} & \text{for } x^* \gg 1 \end{cases} \qquad (7a)$$

or

$$Nux/(GrxPr/16H)^{1/4} \cong \begin{cases} (u_i^{*2}/x^*)^{1/4} & \text{for } x^* \ll 1 \\ \left(\dfrac{\Lambda + 3}{\lambda} \right)^{1/4} & \text{for } x^* \gg 1 \end{cases} \qquad (7b)$$

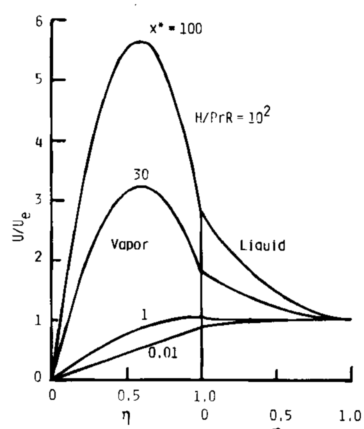

Figure 2. Development of velocity profile.

The shape factors u_i^* and Λ in the above expressions are their asymptotic values, which may readily be calculated from Eqs. 12 and 14b of Part I for a given H/PrR. The asymptotes for these heat transfer groupings (unlike those for u_i^* shown in Figure 1) are quite insensitive to H/PrR. In fact the following asymptotes for the limiting case, namely, $H/PrR = \infty$, may well represent the asymptotes for all cases, provided $H/PrR \gg 1$:

$$Nux/(RexPrR/4H)^{1/2}(\mu_f/\mu) = \begin{cases} 1 & \text{for } x^* \leqq 1 \\ x^{*1/4} & \text{for } x^* \geqq 1 \end{cases} \tag{8a}$$

or

$$Nux/(GrxPr/16H)^{1/4} = \begin{cases} x^{*-1/4} & \text{for } x^* \leqq 1 \\ 1 & \text{for } x^* \geqq 1 \end{cases} \tag{8b}$$

These asymptotes for $H/PrR = \infty$ are also plotted in Figure 3 for the purpose of comparison. The figures indicate that the simple expression given by Eq. 8 are quite useful for estimating heat transfer rates in the film boiling.

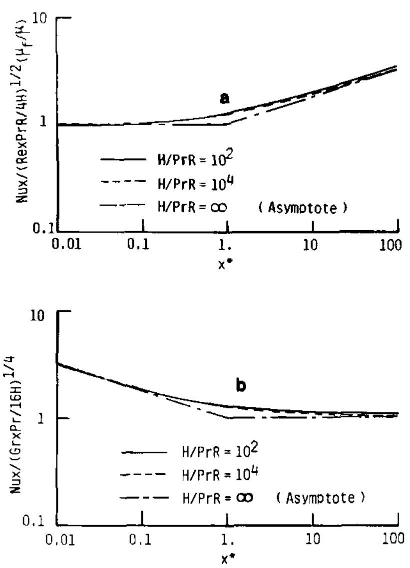

Figure 3. Heat transfer results. (a) Presentation appropriate for forced convection region. (b) Presentation appropriate for free convection region.

CONCLUDING REMARKS

It is particularly interesting to note that the solution of the combined free and forced convection film boiling depends solely on a single lumped parameter H/PrR, when x^* is chosen as the independent variable under the assumption of thin vapor layer. (Note that Jacobs and Boehm, 1970 had to provide three separate parameters, ρ/ρ_f, μ/μ_f, and H/Pr.) The authors believe that the integral formulations such as introduced here, can be used quite effectively not only to study the parametric effects on the film boiling heat transfer, but also to investigate the limiting conditions that are implicit in the governing equations.

NOTATION

Grx	=	Grashof number
H	=	sensible-latent heat ratio
Nux	=	local Nusselt number
Pr	=	Prandtl number of vapor
R	=	density-viscosity ratio
Rex	=	Reynolds number
u	=	streamwise velocity component
u_i, u_i^*	=	interfacial velocity, $u_i^* = u_i/u_e$
x^*	=	dimensionless coordinate defined by Eq. 9b Part I
Λ	=	velocity shape factor defined by Eq. 2c Part I
μ	=	viscosity

Subscripts

e	=	free stream
f	=	liquid

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